Link-Based Classification

►Collective Classification

Liquid State Machine

►Reservoir Computing

Local Distance Metric Adaptation

Synonyms

Supersmoothing; Nonstationary kernels; Kernel shaping

Definition

In learning systems with kernels, the shape and size of a kernel plays a critical role for accuracy and generalization. Most kernels have a distance metric parameter, which determines the size and shape of the kernel in the sense of a Mahalanobis distance. Advanced kernel learning tune every kernel's distance metric individually, instead of turning one global distance metric for all kernels.

Cross References

►Locally Weighted Regression for Control

Local Feature Selection

►Projective Clustering

Locality Sensitive Hashing Based Clustering

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The basic idea of the LSH (Gionis, Indyk, & Motwani, 1999) technique is using multiple hash functions to hash the data points and guarantee that there is a high probability of collision for points which are close to each other and low collision probability for dissimilar points. LSH schemes exist for many distance measures, such as Hamming norm, L_p norms, cosine distance, earth movers distance (EMD), and Jaccard coefficient.

In LSH, define a family $H = \{h : S \to U\}$ as locality-sensitive, if for any a, the function $p(t) = Pr_H[h(a) = h(b) : ||a - b|| = x]$ is decreasing in x. Based on this definition, the probability of collision of points a and b is decreasing with their distance.

Although LSH was originally proposed for approximate nearest neighbor search in high dimensions, it can be used for clustering as well (Das, Datar, Garg, & Rajaram, 2007; Haveliwala, Gionis, & Indyk, 2000). The buckets could be used as the bases for clustering. Seeding the hash functions several times can help getting better quality clustering.

Recommended Reading

Das, A. S., Datar, M., Garg, A., & Rajaram, S. (2007). Google news personalization: Scalable online collaborative filtering. In *WWW '07: Proceedings of the 16th international conference on World Wide Web* (pp. 271–280). New York: ACM.

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Locally Weighted Learning

►Locally Weighted Regression for Control

Locally Weighted Regression for Control

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Synonyms

Kernel shaping; Lazy learning; Local distance metric adaptation; Locally weighted learning; LWPR; LWR; Nonstationary kernels supersmoothing

Definition

This article addresses two topics: ►learning control and locally weighted regression.

►Learning control refers to the process of acquiring a control strategy for a particular control system and a particular task by trial and error. It is usually distinguished from adaptive control (Aström & Wittenmark, 1989) in that the learning system is permitted to fail during the process of learning, resembling how humans and animals acquire new movement strategies. In contrast, adaptive control emphasizes single trial convergence without failure, fulfilling stringent performance constraints, e.g., as needed in life-critical systems like airplanes and industrial robots.

Locally weighted regression refers to ►supervised learning of continuous functions (otherwise known as function approximation or ►regression) by means of spatially localized algorithms, which are often discussed in the context of ►kernel regression, ►nearest neighbor methods, or ►lazy learning (Atkeson, Moore, & Schaal, 1997). Most regression algorithms are global learning systems. For instance, many algorithms can be understood in terms of minimizing a global ►loss function such as the expected sum squared error:

$$J_{\text{global}} = E\left[\frac{1}{2}\sum_{i=1}^{N}\left(\mathbf{t}_i - \mathbf{y}_i\right)^2\right] = E\left[\frac{1}{2}\sum_{i=1}^{N}\left(\mathbf{t}_i - \phi\left(\mathbf{x}_i\right)^T\boldsymbol{\beta}\right)^2\right] \tag{1}$$

where $E[\cdot]$ denotes the expectation operator, $\mathbf{t}_i$ the noise-corrupted target value for an input $\mathbf{x}_i$, which is expanded by basis functions into a basis function vector $\phi(\mathbf{x}_i)$, and $\boldsymbol{\beta}$ the vector of (usually linear) regression coefficients. Classical feedforward ►neural networks, ►radial basis function networks, ►mixture models, or ►Gaussian Process regression are all global function approximators in the spirit of Eq. (1).

In contrast, local learning systems split up conceptually the cost function into multiple independent local function approximation problems, using a cost function such as the one below:

$$\begin{aligned} J_{\text{global}} &= E\left[\frac{1}{2}\sum_{k=1}^{K}\sum_{i=1}^{N} w_{k,i}\left(\mathbf{t}_i - \mathbf{x}_i^T\boldsymbol{\beta}_k\right)^2\right] \\ &= \frac{1}{2}\sum_{k=1}^{K} E\left[\sum_{i=1}^{N} w_{k,i}\left(\mathbf{t}_i - \mathbf{x}_i^T\boldsymbol{\beta}_k\right)^2\right] \end{aligned} \tag{2}$$

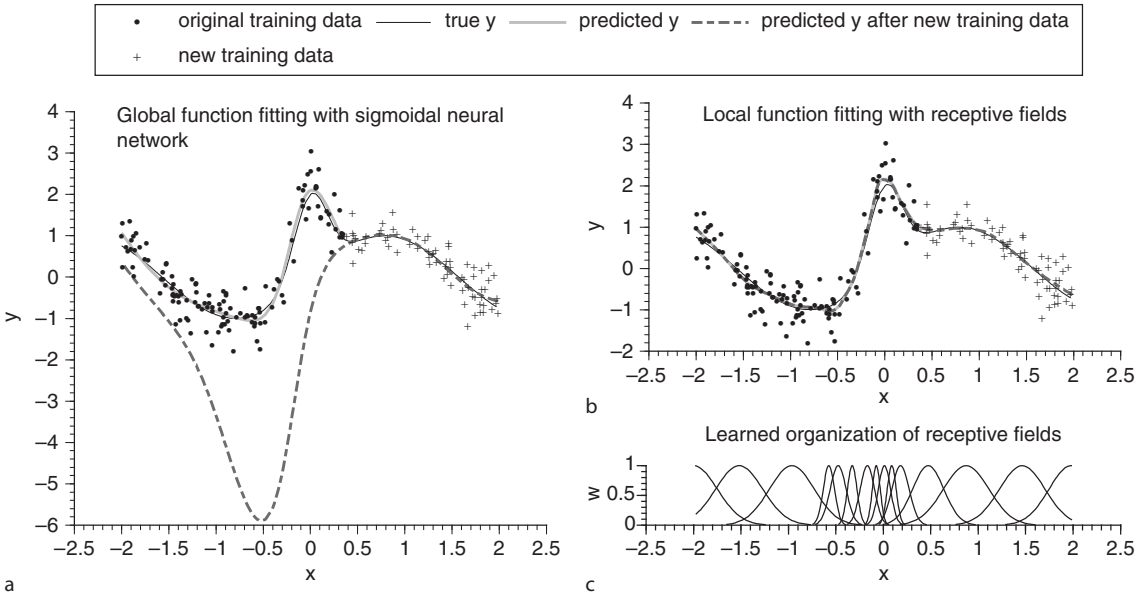
Motivation and Background

Figure 1 illustrates why locally weighted regression methods are often favored over global methods when it comes to learning from incrementally arriving data, especially when dealing with nonstationary input distributions. The figure shows the division of the training data into two sets: the "original training data" and the "new training data" (in dots and crosses, respectively).

Initially, a sigmoidal ►neural network and a locally weighted regression algorithm are trained on the "original training data," using 20% of the data as a cross-validation set to assess convergence of the learning. In a second phase, both learning systems are trained solely on the "new training data" (again with a similar cross-validation procedure), but without using any data from the "original training data." While both algorithms generalize well on the "new training data," the global learner incurred catastrophic interference, unlearning what was learned initially, as seen in Fig. 1a, b shows that the locally weighted regression algorithm does not have this problem since learning (along with ►generalization) is restricted to a local area.

Appealing properties of locally weighted regression include the following:

- Function approximation can be performed incrementally with nonstationary input and output distributions and without significant danger of interference. Locally weighted regression can provide ►posterior probability distributions, offer confidence assessments, and deal with heteroscedastic data.
- Locally weighted learning algorithms are computationally inexpensive to compute. It is well suited for online computations (e.g., for ►online and ►incremental learning) in the fast control loop of a robot – typically on the order of 100–1000 Hz.
- Locally weighted regression methods can implement continual learning and learning from large amounts of data without running into severe computational problems on modern computing hardware.
- Locally weighted regression is a nonparametric method (i.e., it does not require that the user determine *a priori* the number of local models in the learning system), and the learning systems grows with the complexity of the data it tries to model.
- Locally weighted regression can include ►feature selection, ►dimensionality reduction, and ►Bayesian inference – all which are required for robust statistical inference.



Locally Weighted Regression for Control. Figure 1. Function approximation results for the function $y = \sin(2x) + 2\exp(-16x^2) + N(0, 0.16)$ with (a) a sigmoidal neural network; (b) a locally weighted regression algorithm (note that the data traces "true y," "predicted y," and "predicted y after new training data" largely coincide); and (c) the organization of the (Gaussian) kernels of (b) after training. See Schaal and Atkeson (1998) for more details

- Locally weighted regression works favorably with locally linear models (Hastie & Loader, 1993), and local linearizations are of ubiquitous use in control applications.

Background

Returning to Eqs. (1) and (2), the main differences between both equations are listed below:

(i) A weight $w_{i,k}$ is introduced that focuses the function approximation on only a small neighborhood around a point of interest $\mathbf{c}_k$ in input space (see Eq. 3 below).
(ii) The cost function is split into K independent optimization problems.
(iii) Due to the restricted scope of the function approximation problem, we do not need a nonlinear basis function expansion and can, instead, work with simple local functions or local polynomials (Hastie & Loader, 1993).

The weights $w_{k,i}$ in Eq. (2) are typically computed from some ▶kernel function (Atkeson, Moore, & Schaal, 1997) such as a squared exponential kernel

$$w_{k,i} = \exp\left(-\frac{1}{2}(\mathbf{x}_i - \mathbf{c}_k)^T \mathbf{D}_k (\mathbf{x}_i - \mathbf{c}_k)\right) \quad (3)$$

with $\mathbf{D}_k$ denoting a positive semidefinite distance metric and $\mathbf{c}_k$ the center of the kernel. The number of kernels K is not finite. In many local learning algorithms, the kernels are never maintained in memory. Instead, for every query point $\mathbf{x}_q$, a new kernel is centered at $\mathbf{c}_k = \mathbf{x}_q$, and the localized function approximation is solved with weighted ▶regression techniques (Atkeson et al., 1997).

Locally weighted regression should not be confused with mixture of experts models (Jordan & Jacobs, 1994). ▶Mixture models are *global* learning systems since the experts compete globally to cover training data. Mixture models address the ▶bias-variance dilemma (Intuitively, the ▶bias-variance dilemma addresses how many parameters to use for a function approximation problem to find an optimal balance between ▶overfitting and oversmoothing of the training data) by finding the right number of local experts. Locally weighted regression addresses the ▶bias-variance dilemma in a local way by finding the

optimal distance metric for computing the weights in the locally weighted regression (Schaal & Atkeson, 1998). We describe some algorithms to find $\mathbf{D}_k$ next.

Structure of Learning System

For a locally linear model centered at the query point $\mathbf{x}_q$, the regression coefficients would be

$$\boldsymbol{\beta}_q = \left(\mathbf{X}^T\mathbf{W}_q\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{W}_q\mathbf{t} \tag{4}$$

where $\mathbf{X}$ is a matrix that has all training input data points in its rows (with a column of 1s added in the last column for the offset parameter in ►linear regression). $\mathbf{W}_q$ is a diagonal matrix with the corresponding weights for all data points, computed from Eq. (3) with $\mathbf{c}_k = \mathbf{x}_q$, and $\mathbf{t}$ is the vector of regression targets for all training points. Such a "compute-the-prediction-on-the-fly" approach is often called lazy learning (The approach is "lazy" because the computational of a prediction is deferred until the last moment, i.e., when a prediction is needed) and is a memory-based learning system where all training data is kept in memory for making predictions.

Alternatively, kernels can be created as needed to cover the input space, and the sufficient statistics of the weighted regression are updated incrementally with recursive least squares (Schaal & Atkeson, 1998). This approach does not require storage of data points in memory. Predictions of neighboring local models can be blended, improving function fitting results in the spirit of committee machines.

Memory-Based Locally Weighted Regression (LWR)

The original locally weighted regression algorithm was introduced by Cleveland (1979) and popularized in the machine learning and learning control community by Atkeson (1989). The algorithm is largely summarized by Eq. (4) (for algorithmic pseudo-code, see (Schaal, Atkeson, & Vijayakumar, 2002)):

- All training data is collected in the matrix $\mathbf{X}$ and the vector $\mathbf{t}$ (For simplicity, only functions with a scalar output are addressed. Vector-valued outputs can be learned either by fitting a separate learning system for each output or by modifying the algorithms to fit multiple outputs (similar to multi-output linear regression)).
- For every query point $\mathbf{x}_q$, the weighting kernel is centered at the query point.
- The weights are computed with Eq. (3).
- The local regression coefficients are computed according to Eq. (4).
- A prediction is formed with $y_q = \left[\mathbf{x}_q^T\ 1\right]\boldsymbol{\beta}_q$.

As in all kernel methods, it is important to optimize the kernel parameters in order to get optimal function fitting quality. For LWR, the critical parameter determining the ►bias-variance tradeoff is the distance metric $\mathbf{D}_q$. If the kernel is too narrow, it starts fitting noise. If it is too broad, oversmoothing will occur. $\mathbf{D}_q$ can be optimized with leave-one-out cross-validation to obtain a *globally* optimal value, i.e., the same $\mathbf{D}_q = \mathbf{D}$ is used throughout the entire input space of the data. Alternatively, $\mathbf{D}_q$ can be *locally* optimized as a function of the query point, i.e., obtain a $\mathbf{D}_q$ as a function of the query point (as already indicated by the subscript "q"). In the recent machine learning literature (in particular, work related to kernel methods), such input dependent kernels are referred to as nonstationary kernels.

Locally Weighted Projection Regression (LWPR)

Schaal and Atkeson (1998) suggested a memoryless version of LWR in order to avoid the expensive ►nearest neighbor computations – particularly for large training data sets – of LWR and to have fast real-time (In most robotic systems, "real-time" means on the order of maximally 1–10 ms computation time, corresponding to a 1000–100 Hz control loop) prediction performance. The main ideas of the RFWR algorithm (Schaal & Atkeson, 1998) are listed below:

- Create new kernels only if no existing kernel in memory covers a training point with some minimal activation weight.
- Keep all created kernels in memory and update the weighted regression with weighted recursive least squares for new training points $\{\mathbf{x}, t\}$:

$$\begin{aligned}\boldsymbol{\beta}_k^{n+1} &= \boldsymbol{\beta}_k^n + w\mathbf{P}^{n+1}\tilde{\mathbf{x}}\left(t - \tilde{\mathbf{x}}^T\boldsymbol{\beta}_k^n\right)\\ \text{where } \mathbf{P}_k^{n+1} &= \frac{1}{\lambda}\left(\mathbf{P}_k^n - \frac{\mathbf{P}_k^n\tilde{\mathbf{x}}\tilde{\mathbf{x}}^T\mathbf{P}_k^n}{\frac{\lambda}{w} + \tilde{\mathbf{x}}^T\mathbf{P}_k^n\tilde{\mathbf{x}}}\right)\\ \text{and } \tilde{\mathbf{x}} &= \left[\mathbf{x}^T\ 1\right]^T.\end{aligned} \tag{5}$$

- Adjust the distance metric $\mathbf{D}_q$ for each kernel with a gradient descent technique using leave-one-out cross-validation.
- Make a prediction for a query point taking a weighted average of predictions from all local models:

$$\mathbf{y}_q = \frac{\sum_{k=1}^{K} w_{q,k} \hat{\mathbf{y}}_{q,k}}{\sum_{k=1}^{K} w_{q,k}} \tag{6}$$

Adjusting the distance metric $\mathbf{D}_q$ with leave-one-out cross-validation *without* keeping all training data in memory is possible due to the PRESS residual. The PRESS residual allows the leave-one-out cross-validation error to be computed in closed form without needing to actually exclude a data point from the training data.

Another deficiency of LWR is its inability to scale well to high-dimensional input spaces since the ►covariance matrix inversion in Eq. (4) becomes severely ill-conditioned. Additionally, LWR becomes expensive to evaluate as the number of local models to be maintained increases. Vijayakumar, D'Souza and Schaal (2005) suggested local ►dimensionality reduction techniques to handle this problem. Partial least squares (PLS) regression is a useful ►dimensionality reduction method that is used in the LWPR algorithm (Vijayakumar et al., 2005). In contrast to PCA methods, PLS performs ►dimensionality reduction for ►regression, i.e., it eliminates subspaces of the input space that minimally correlate with the outputs, not just parts of the input space that have low variance.

LWPR is currently one of the best developed locally weighted regression algorithms for control (Klanke, Vijayakumar, & Schaal, 2008) and has been applied to learning control problems with over 100 input dimensions.

A Full Bayesian Treatment of Locally Weighted Regression

Ting, Kalakrishnan, Vijayakumar, and Schaal (2008) proposed a fully probabilistic treatment of LWR in an attempt to avoid cross-validation procedures and minimize any manual parameter tuning (e.g., gradient descent rates, kernel initialization, and forgetting rates). The resulting Bayesian algorithm learns the distance metric of local linear model (For simplicity, a local linear model is assumed, although local polynomials can be used as well) probabilistically, can cope with high input dimensions, and rejects data outliers automatically. The main ideas of Bayesian LWR are listed below (please see Ting (2009) for details):

- Introduce hidden variables $\mathbf{z}$ to the local linear model (as in Variational Bayesian least squares (Ting et al., 2005)) to decompose the statistical estimation problem into d individual estimation problems (where d is the number of input dimensions). The result is an iterative Expectation-Maximization (EM) algorithm that is of linear ►computational complexity in d and the number of training data samples N, i.e., $O(Nd)$.
- Associate a scalar weight w_i with each training data sample $\{\mathbf{x}_i, t_i\}$, placing a Bernoulli ►prior probability distribution over a weight *for each input dimension* so that the weights are positive and between 0 and 1:

$$w_i = \prod_{m=1}^{d} w_{im} \text{ where}$$
$$w_{im} \sim \text{Bernoulli}\,(q_{im}) \text{ for } i = 1, .., N; m = 1, .., d \tag{7}$$

where the weight w_i is decomposed into independent components in each input dimension w_{im} and q_{im} is the parameter of the Bernoulli ►probability distribution. The weight w_i indicates a training sample's contribution to the local model. An outlier will have a weight of 0 and will, thus, be automatically rejected. The formulation of q_{im} determines the shape of the weighting function applied to the local model. The weighting function q_{im} used in Bayesian LWR is listed below:

$$q_{im} = \frac{1}{1 + \left(x_{im} - x_{qm}\right)^2 h_m} \text{ for } i = 1, .., N; m = 1, .., d \tag{8}$$

where $\mathbf{x}_q \in \Re^{d \times 1}$ is the query input point and h_m is the bandwidth parameter/distance metric of the local model in the m-th input dimension (The distance metric/bandwidth is assumed to be a diagonal matrix, i.e., bandwidths in each input dimension are independent. That is to say, $\mathbf{D} = \mathbf{H}$, where $\mathbf{h}$ is the diagonal vector and h_m are the coefficients of $\mathbf{h}$).

- Place a Gamma ▶prior probability distribution over the distance metric h_m:

$$h_m \sim \text{Gamma}\,(a_{hm0}, b_{hm0}) \tag{9}$$

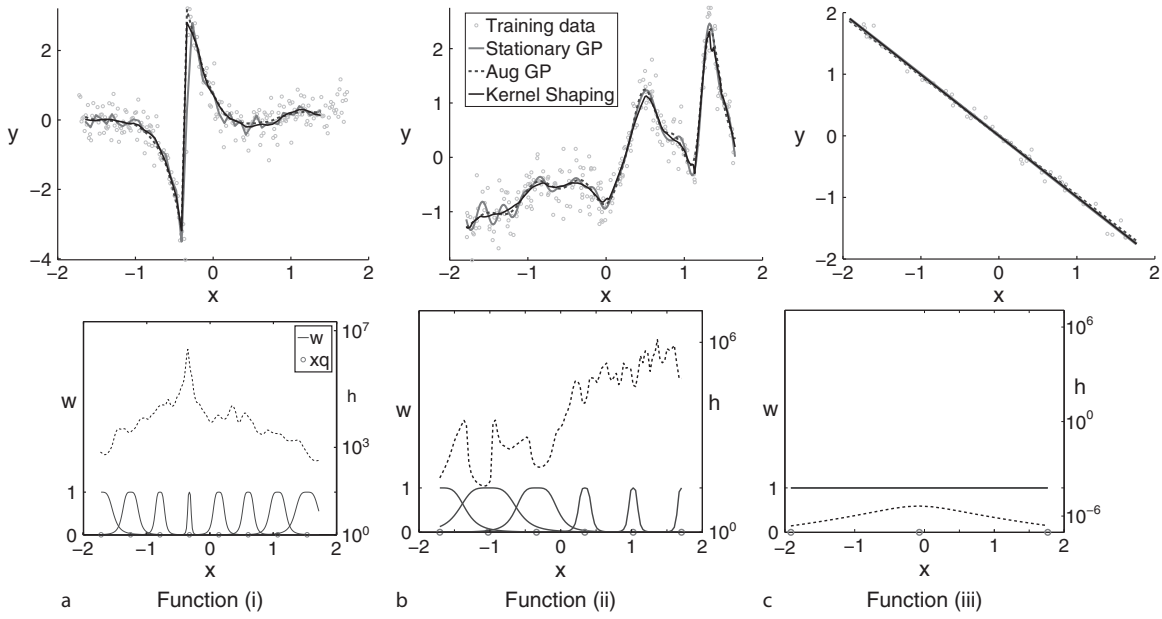
where $\{a_{hm0}, b_{hm0}\}$ are the prior parameter values of the Gamma distribution.
- Treat the model as an EM-like ▶regression problem, using ▶variational approximations to achieve analytically tractable inference of the ▶posterior probability distributions.

The initial parameters $\{a_{hm0}, b_{hm0}\}$ should be set so that the ▶prior probability distribution over h_m is uninformative and wide (e.g., $a_{hm0} = b_{hm0} = 10^{-6}$). The other ▶prior probability distribution that needs to be specified is the one over the noise variance random variable – and this is best set to reflect how noisy the data set is believed to be. More details can be found in Ting (2009).

This Bayesian method can can also be applied as general kernel shaping algorithm for global ▶kernel learning methods that are linear in the parameters (e.g., to realize nonstationary ▶Gaussian processes (Ting et al., 2008), resulting in an augmented nonstationary ▶Gaussian Process).

Figure 2 illustrates Bayesian kernel shaping's bandwidth adaptation abilities on several synthetic data sets, comparing it to a stationary ▶Gaussian Process and the augmented nonstationary ▶Gaussian Process. For the ease of visualization, the following one-dimensional functions are considered: (i) a function with a discontinuity, (ii) a spatially inhomogeneous function, and (iii) a straight line function. The data set for function (i) consists of 250 training samples, 201 test inputs (evenly spaced across the input space), and output noise with variance of 0.3025; the data set for function (ii) consists of 250 training samples, 101 test inputs, and an output signal-to-noise ratio (SNR) of 10; and the data set for function (iii) has 50 training samples, 21 test inputs, and an output SNR of 100. Figure 2 shows the predicted outputs of all three algorithms for data sets (i)–(iii). The local kernel shaping algorithm smoothes over regions where a stationary ▶Gaussian Process overfits and yet, it still manages to capture regions of highly varying curvature, as seen in Figs. 2a and 2b.

It correctly adjusts the bandwidths h with the curvature of the function. When the data looks linear, the algorithm opens up the weighting kernel so that all data samples are considered, as Fig. 2c shows.



Locally Weighted Regression for Control. Figure 2. Predicted outputs using a stationary Gaussian Process (GP), the augmented nonstationary GP and local kernel shaping on three different data sets. Figures on the bottom row show the bandwidths learned by local kernel shaping and the corresponding weighting kernels (*in dotted black lines*) for various input query points (*shown in red circles*)

From the viewpoint of ▶learning control, ▶overfitting – as seen in the ▶Gaussian Process in Fig. 2 – can be detrimental since ▶learning control often relies on extracting local linearizations to derive ▶controllers (see Applications section). Obtaining the wrong sign on a slope in a local linearization may destabilize a ▶controller.

In contrast to LWPR, the Bayesian LWR method is memory-based, although memoryless versions could be derived. Future work will also have to address how to incorporate ▶dimensionality reduction methods for robustness in high dimensions. Nevertheless, it is a first step toward a probabilistic locally weighted regression method with minimal parameter tuning required by the user.

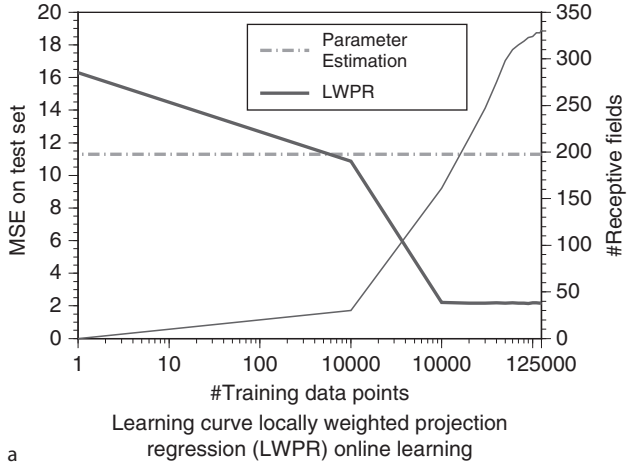
Applications

Learning Internal Models with LWPR

Learning an internal model is one of most typical applications of LWR methods for control. The model could be a forward model (e.g, the nonlinear differential equations of robot dynamics), an inverse model (e.g., the equations that predict the amount of torque to achieve a change of state in a robot), or any other function that models associations between input and output data about the environment. The models are used, subsequently, to compute a ▶controller e.g., an inverse dynamics controller similar to Eq. (12). Models for complex robots such as humanoids exceed easily a hundred input dimensions. In such high-dimensional spaces, it is hopeless to assume that a representative data set can be collected for offline training that can generalize sufficiently to related tasks. Thus, the LWR philosophy involves having a learning algorithm that can learn rapidly when entering a new part of the state space such that it can achieve acceptable ▶generalization performance almost instantaneously.

Figure 3 demonstrates ▶online learning of an inverse dynamics model for the elbow joint (cf. Eq. 12) for a Sarcos Dexterous Robot Arm. The robot starts with no knowledge about this model, and it tracks some randomly varying desired trajectories with a proportional-derivative (PD) controller. During its movements, training data consisting of tuples $(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \tau)$ – which model a mapping from joint position, joint velocities and joint accelerations $(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$ to motor torques τ – are collected (at about every 2 ms). Every data point is used to train a LWPR function approximator, which generates a feedforward command for the controller. The ▶learning curve is shown in Fig. 3a.

Using a test set created beforehand, the model predictions of LWPR are compared every 1,000 training points with that of a parameter estimation method. The parameter estimation approach fits the minimal number of parameters to an analytical model of the robot dynamics under an idealized rigid body dynamics (RBD) assumptions, using all training data (i.e., not incrementally). Given that the Sarcos robot is a



a Learning curve locally weighted projection regression (LWPR) online learning

b Seven Degree-of-Freedom Sarcos Robot Arm

Locally Weighted Regression for Control. Figure 3. Learning an inverse dynamics model in real-time with a high-performance anthropomprohic robot arm

hydraulic robot, the RBD assumption is not very suitable, and, as Fig. 3a shows, LWPR (in thick red line) outperforms the analytical model (in dotted blue line) after a rather short amount of training. After about 5 min of training (about 125,000 data points), very good performance is achieved, using about 350 local models. This example demonstrates (i) the quality of function approximation that can be achieved with LWPR and (ii) the online allocation of more local models as needed.

Learning Paired Inverse-Forward Models

Learning inverse models (such as inverse kinematics and inverse dynamics models) can be challenging since the inverse model problem is often a relation, not a function, with a one-to-many mapping. Applying any arbitrary nonlinear function approximation method to the inverse model problem can lead to unpredictably bad performance, as the training data can form non-convex solution spaces, in which averaging is inappropriate. Architectures such as ►mixture models (in particular, mixture density networks) have been proposed to address problems with non-convex solution spaces. A particularly interesting approach in control involves learning linearizations of a forward model (which is proper function) and learning an inverse mapping within the local region of the forward model.

Ting et al. (2008) demonstrated such a forward-inverse model learning approach with Bayesian LWR to learn an inverse kinematics model for a haptic robot arm (shown in Fig. 4) in order to control the end-effector along a desired trajectory in task space. Training data was collected while the arm performed random sinusoidal movements within a constrained box volume of Cartesian space. Each sample consists of the arm's joint angles $\mathbf{q}$, joint velocities $\dot{\mathbf{q}}$, end-effector position in Cartesian space $\mathbf{x}$, and end-effector velocities $\dot{\mathbf{x}}$. From this data, a forward kinematics model is learned:

$$\dot{\mathbf{x}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} \tag{10}$$

where $\mathbf{J}(\mathbf{q})$ is the Jacobian matrix. The transformation from $\dot{\mathbf{q}}$ to $\dot{\mathbf{x}}$ can be assumed to be locally linear at a particular configuration $\mathbf{q}$ of the robot arm. Bayesian LWR is used to learn the forward model, and, as in LWPR, local models are only added if a training point is not already sufficiently covered by an existing local model. Importantly, the kernel functions in LWR are localized only with respect to $\mathbf{q}$, while the regression of each model is trained only on a mapping from $\dot{\mathbf{q}}$ to $\dot{\mathbf{x}}$ – these geometric insights are easily incorporated as priors in Bayesian LWR, as they are natural to locally linear models. Incorporating these priors in other function approximators, e.g., ►Gaussian Process regression, is not straightforward.

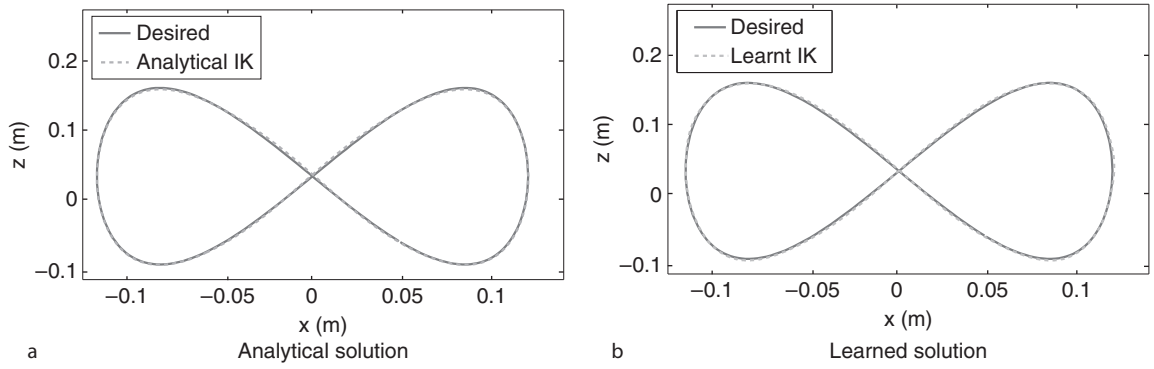
The goal of the robot task is to track a desired trajectory $(\mathbf{x}, \dot{\mathbf{x}})$ specified only in terms of x, z positions and velocities, i.e., the movement is supposed to be in a vertical plane in front of the robot, but the exact position of the vertical plane is not given. Thus, the task has one degree of redundancy, and the learning system needs to generate a mapping from $\{\mathbf{x}, \dot{\mathbf{x}}\}$ to $\dot{\mathbf{q}}$. Analytically, the inverse kinematics equation is

$$\dot{\mathbf{q}} = \mathbf{J}^{\#}(\mathbf{q})\dot{\mathbf{x}} - \alpha(\mathbf{I} - \mathbf{J}^{\#}\mathbf{J})\frac{\partial g}{\partial \mathbf{q}} \tag{11}$$

where $J^{\#}(\mathbf{q})$ is the pseudo-inverse of the Jacobian. The second term is an gradient descent optimization term for redundancy resolution, specified here by a cost function g in terms of joint angles $\mathbf{q}$.

To learn an inverse kinematics model, the local regions of $\mathbf{q}$ from the forward model can be re-used since any inverse of $\mathbf{J}$ is locally linear within these regions. Moreover, for locally linear models, all solution spaces for the inverse model are locally convex, such that an inverse can be learned without problems. The redundancy issue can be solved by applying an additional weight to each data point according to a reward function. Since the experimental task is specified in

Locally Weighted Regression for Control. Figure 4. SensAble Phantom haptic robotic arm



Locally Weighted Regression for Control. Figure 5. Desired versus actual trajectories for SensAble Phantom robot arm

terms of $\{\dot{x}, \dot{z}\}$, a reward is defined, based on a desired y coordinate, y_{des}, and enforced as a soft constraint. The resulting reward function, is $g = e^{-\frac{1}{2}h(k(y_{des}-y)-\dot{y})^2}$, where k is a gain and h specifies the steepness of the reward. This ensures that the learned inverse model chooses a solution that pushes $\dot{y}$ toward y_{des}. Each forward local model is inverted using a weighted ▶linear regression, where each data point is weighted by the kernel weight from the forward model and additionally weighted by the reward. Thus, a piecewise locally linear solution to the inverse problem can be learned efficiently.

Figure 5 shows the performance of the learned inverse model (Learnt IK) in a figure-eight tracking task. The learned model performs as well as the analytical inverse kinematics solution (Analytical IK), with root mean squared tracking errors in positions and velocities very close to that of the analytical solution.

Learning Trajectory Optimizations

Mitrovic, Klanke, and Vijayakumar (2008) have explored a theory for sensorimotor adaptation in humans, i.e., how humans replan their movement trajectories in the presence of perturbations. They rely on the iterative Linear Quadratic Gaussian (iLQG) algorithm (Todorov & Li, 2004) to deal with the nonlinear and changing plant dynamics that may result from altered morphology, wear and tear, or external perturbations. They take advantage of the "on-the-fly" adaptation of locally weighted regression methods like LWPR to learn the forward dynamics of a simulated arm for the purpose of optimizing a movement trajectory between a start point and an end point.

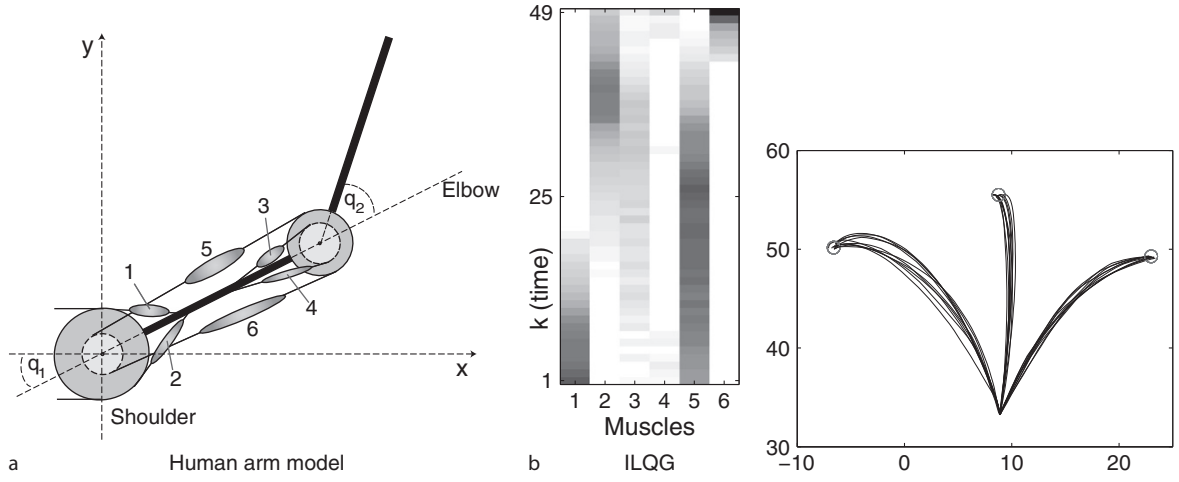
Figure 6a shows the diagram of a two degrees-of-freedom planar human arm model, which is actuated by four single-joint and two double-joint antagonistic muscles. Although kinematically simple, the system is over-actuated and, therefore, it is an interesting testbed because large redundancies in the dynamics have to be resolved. The dimensionality of the control signals makes adaptation processes (e.g., to external force fields) quite demanding.

The dynamics of the arm is, in part, based on standard RBD equations of motion:

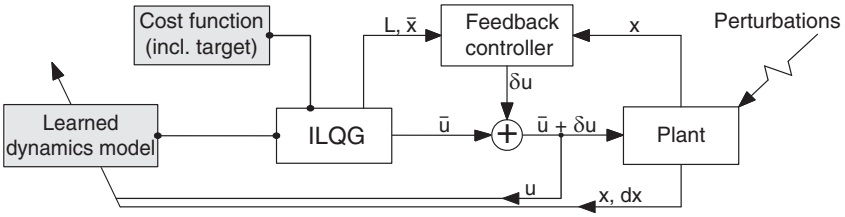
$$\tau = \mathbf{M}(\mathbf{q})\,\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\,\dot{\mathbf{q}} \tag{12}$$

where τ are the joint torques; $\mathbf{q}$ and $\dot{\mathbf{q}}$ are the joint angles and velocities, respectively; $\mathbf{M}(\mathbf{q})$ is the two-dimensional symmetric joint space inertia matrix; and $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ accounts for Coriolis and centripetal forces. Given the antagonistic muscle-based actuation, it is not possible to command joint torques directly. Instead, the effective torques from the muscle activations $\mathbf{u}$ – which happens to be quadratic in $\mathbf{u}$ – should be used. As a result, in contrast to standard torque-controlled robots, the dynamics equation in Eq. (12) is *nonlinear in the control signals* $\mathbf{u}$.

The iLQG algorithm (Todorov & Li, 2004) is used to calculate solutions to "localized" linear and quadratic approximations, which are iterated to improve the global control solution. However, it relies on an analytical forward dynamics model $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$ and finite difference methods to compute gradients. To alleviate this requirement and to make iLQG adaptive, LWPR can be used to learn an approximation of the plant's forward dynamics model. Figure 7 shows the control



Locally Weighted Regression for Control. Figure 6. (a) Human arm model with 6 muscles; (b) Optimized control sequence (*left*) and resulting trajectories (*right*) using the known analytic dynamics model. The control sequences (left target only) for each muscle (1–6) are drawn from bottom to top, with darker grey levels indicating stronger muscle activation

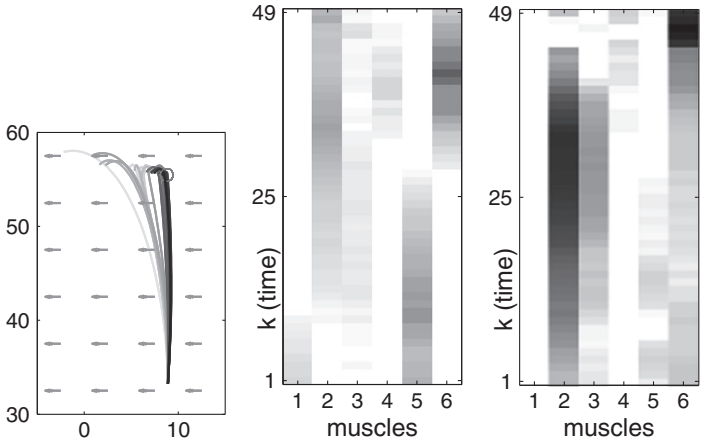


Locally Weighted Regression for Control. Figure 7. Illustration of learning and control scheme of the iterative Linear Quadratic Gaussian (iLQG) algorithm with learned dynamics

diagram, where the "learned dynamics model" (the forward model learned by LWPR) is then updated in an online fashion with every iteration to cope with changes in dynamics. The resulting framework is called iLQG-LD (iLQG with learned dynamics).

Movements of the arm model in Fig. 6a are studied for fixed time horizon reaching movement. The manipulator starts at an initial position $\mathbf{q}_0$ and reaches towards a target $\mathbf{q}_{tar}$. The cost function to be optimized during the movement is a combination of target accuracy and amount of muscle activation (i.e., energy consumption). Figure 6b shows trajectories of generated movements for three reference targets (shown in red circles) using the feedback controller from iLQG with the analytical plant dynamics. The trajectories generated with iLQG-LD (where the forward plant dynamics are learned with LWPR) are omitted as they are hardly distinguishable from the analytical solution.

A major advantage of iLQG-LD is that it does not rely on an accurate analytic dynamics model; this enables the framework to predict adaptation behavior under an ideal observer planning model. Reaching movements were studied where a constant unidirectional force field acting perpendicular to the reaching movement was generated as a perturbation (see Fig. 8 (*left*)). Using the iLQG-LD model, the manipulator gets strongly deflected when reaching for the target because the learned dynamics model cannot yet account for the "spurious" forces. However, when the deflected trajectory is used as training data and the dynamics model is updated online, the tracking improves with each new successive trial (Fig. 8 (*left*)). Please refer to Mitrovic et al. (2008) for more details. Aftereffects upon removing the force field, very similar to those observed in human experiments, are also observed.



Locally Weighted Regression for Control. Figure 8. Adaptation to a unidirectional constant force field (indicated by the arrows). Darker lines indicate better trained models. In particular, the left-most trajectory corresponds to the "initial" control sequence, which was calculated using the LWPR model *before* the adaptation process. The fully "adapted" control sequence results in a nearly straight line reaching movement

Cross References

►Bias and Variance
►Dimensionality Reduction
►Incremental Learning
►Kernel Function
►Kernel Methods
►Lazy Learning
►Linear Regression
►Mixture Models
►Online Learning
►Overfitting
►Radial Basis Functions
►Regression
►Supervised Learning

Programs and Data

http://www-clmc.usc.edu/software
http://www.ipab.inf.ed.ac.uk/slmc/software/

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Logic of Generality

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Synonyms

Generality and logic; Induction as inverted deduction; Inductive inference rules; Is more general than; Is more specific than; Specialization

Definition

One hypothesis is *more general* than another one if it covers all instances that are also covered by the latter one. The former hypothesis is called a ▶*generalization* of the latter one, and the latter a ▶*specialization* of the former. When using logical formulae as hypotheses, the generality relation coincides with the notion of logical entailment, which implies that the generality relation can be analyzed from a logical perspective. The logical analysis of generality, which is pursued in this chapter, leads to the perspective of *induction as the inverse of deduction*. This forms the basis for an analysis of various logical frameworks for reasoning about generality and for traversing the space of possible hypotheses. Many of these frameworks (such as for instance, *θ-subsumption*) are employed in the field of ▶inductive logic programming and are introduced below.

Motivation and Background

Symbolic machine learning methods typically learn by searching a hypothesis space. The hypothesis space can be (partially) ordered by the ▶generality relation, which serves as the basis for defining operators to traverse the space as well as for pruning away unpromising parts of the search space. This is often realized through the use of ▶refinement operators, that is, generalization and specialization operators. Because many learning methods employ a ▶hypothesis language that is logical or that can be reformulated in logic, it is interesting to analyze the generality relation from a logical perspective. When using logical formulae as hypotheses, the generality relation closely corresponds to logical entailment. This allows us to directly transfer results from logic to a machine learning context. In particular, machine learning operators can be derived from logical inference rules. The logical theory of generality provides a framework for transferring these results. Within the standard setting of inductive logic programming, learning from entailment, specialization is realized through deduction, and generalization through induction, which is considered to be the inverse of deduction. Different deductive inference rules lead to different frameworks for generalization and specialization. The most popular one is that of θ-subsumption, which is employed by the vast majority of contemporary inductive logic programming systems.

Theory

A hypothesis g is *more general than* a hypothesis s if and only if g covers all instances that are also covered by s, more formally, if $\text{covers}(s) \subseteq \text{covers}(g)$, in which case, $\text{covers}(h)$ denotes the set of all instances covered by the hypothesis h.

There are several possible ways to represent hypotheses and instances in logic (De Raedt, 1997, 2008), each of which results in a different setting with a corresponding covers relation. Some of the best known settings are *learning from entailment*, learning from interpretations, and learning from proofs.

Learning from Entailment

In learning from entailment, both hypotheses and instances are logical formulae, typically *definite clauses*, which underlie the programming language Prolog (Flach, 1994). Furthermore, when learning from entailment, a hypothesis h covers an instance e if and only if $h \models e$, that is, when h logically entails e, or equivalently, when e is a logical consequence of h. For instance, consider the hypothesis h:

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flies :- bird, normal.
bird :- blackbird.
bird :- ostrich.
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